Overcoming Probabilistic Faults in Disoriented Linear Search

Department of Mathematics

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Joint work with **Nikos Giachoudis and Evangelos Kranakis.**

Presentation Outline

- **Linear Search – Related Work**
- New Problem & Main Results
- Related Work
- Searching with 1 Faulty Agent: Results Outline
- Searching with 2 Faulty Agents: Results Outline
- Future Directions

Deterministic Linear Search – 1 Searcher

Specs:

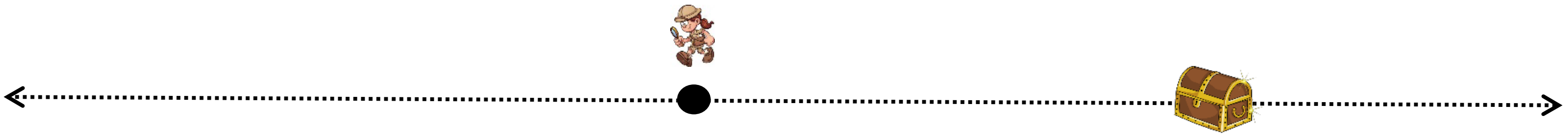
- **One** speed-1 searcher, starting at origin of infinite line.
- No turning cost, no extra time for changing direction.
- Treasure (exit) at unknown location, at least 1 away.
- Treasure cannot be seen from distance.
- Treasure detected when searcher walks over it.

Feasible Solutions:

- **Deterministic** trajectory covering (eventually) entire line.

The objective:

- Minimize "worst case" relative search time ($\frac{time\ to\ hit\ treasure\ d{-}away}{d}$)



Known (Opt) Competitive Ratios

- Optimal deterministic
  0-faulty, 1 searcher
  [Baeza-Yates, Culberson, Rawlins, 1993]

Randomized Linear Search – 1 Searcher

Specs:

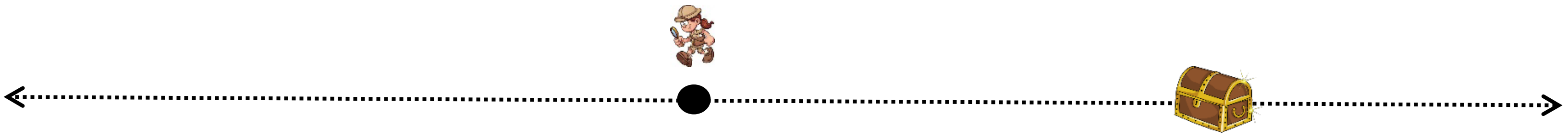
- **One** speed-1 searcher, starting at origin of infinite line.
- No turning cost, no extra time for changing direction.
- Treasure (exit) at unknown location, at least 1 away.
- Treasure cannot be seen from distance.
- Treasure detected when searcher walks over it.

Feasible Solutions:

- **Randomized** trajectory covering (eventually) entire line.

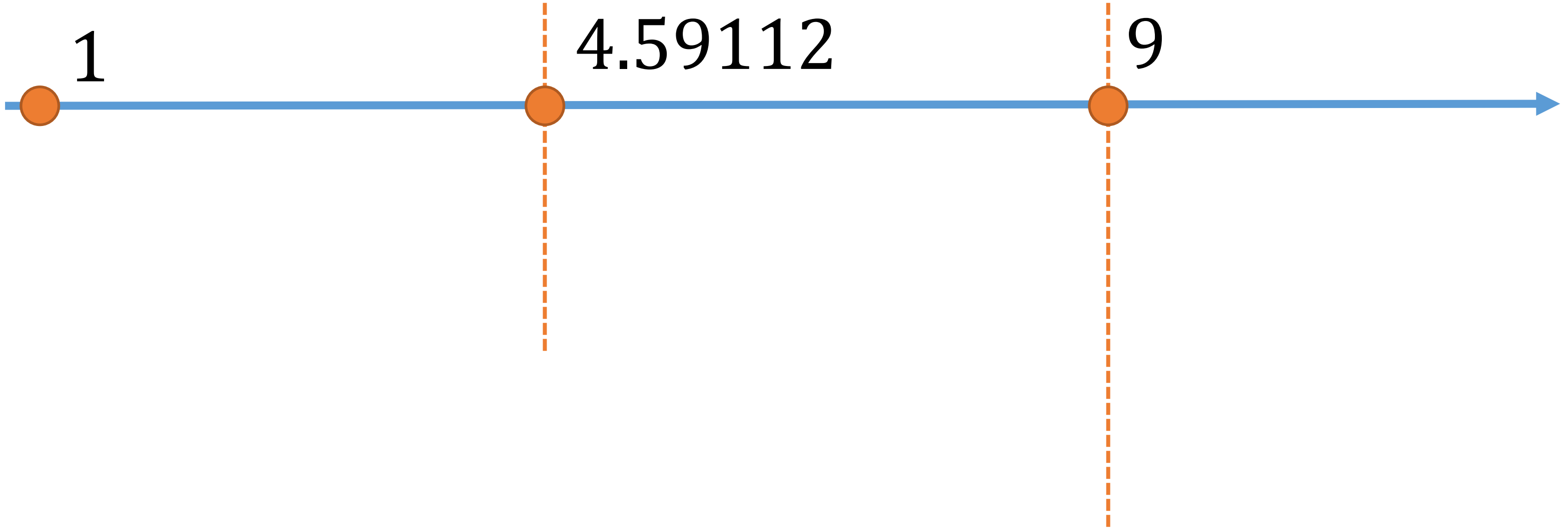
The objective:

- Minimize "worst case" **expected** relative search time



Known (Opt) Competitive Ratios

- Optimal randomized 0-faulty, 1 searcher [Kao, Reif, Tate, 1996]
- Optimal deterministic 0-faulty, 1 searcher



Deterministic Linear Search – 2 Searchers

Specs:

- **Two** speed-1 searchers, starting at origin of infinite line.
- No turning cost, no extra time for changing direction.
- Treasure (exit) at unknown location, at least 1 away.
- Treasure cannot be seen from distance.
- Treasure detected when searcher walks over it.
- **Searchers aware of each others' trajectories (centralized model)**
- **Searchers access to same clock (synchronous model)**
- **Communication between Searchers: F2F or Wireless**

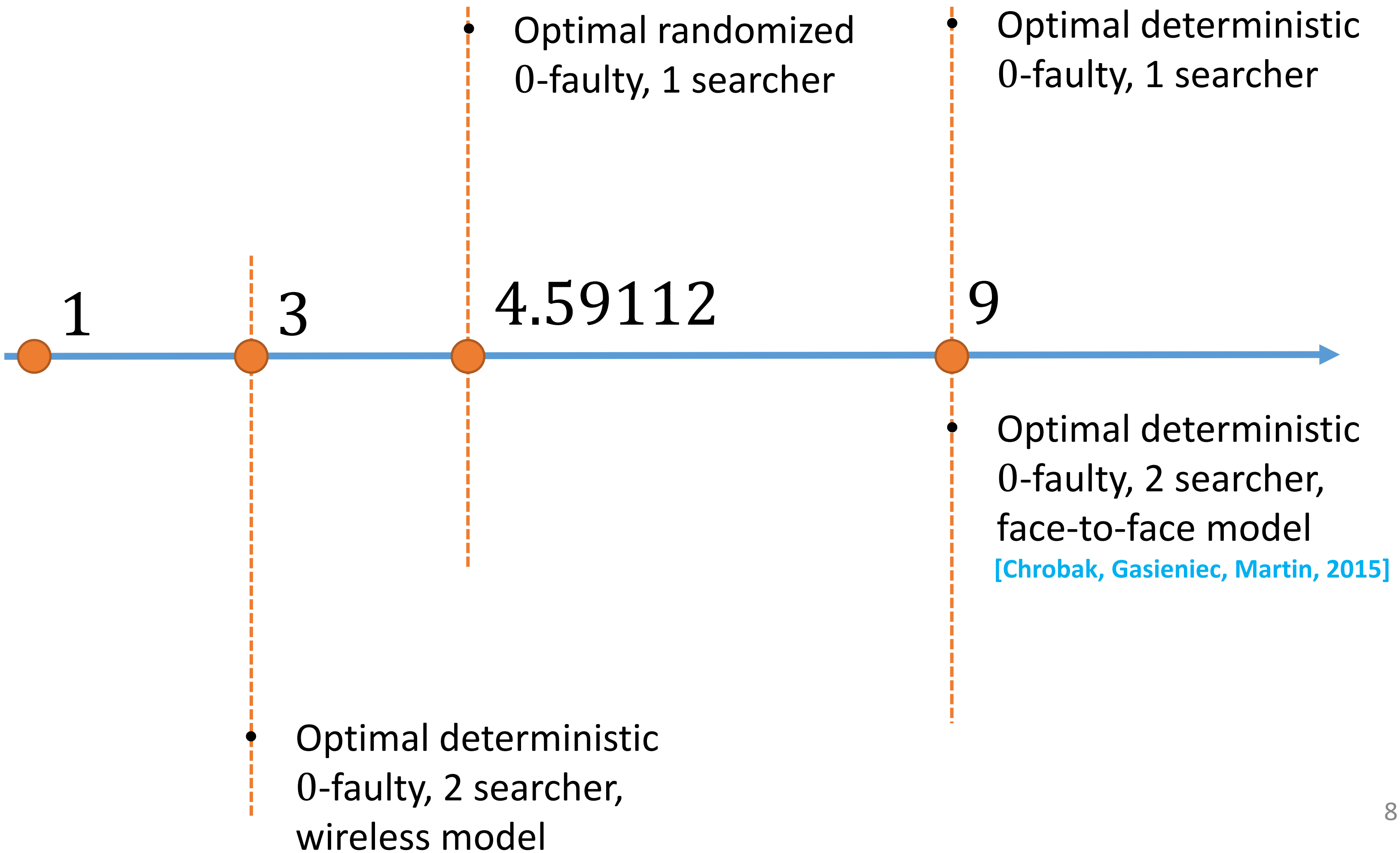
Feasible Solutions:

- **Deterministic** trajectories covering (eventually) entire line.

The objective:

- Minimize "worst case" relative **evacuation** time ($\frac{time\ to\ hit\ treasure\ d{-}away}{d}$)

Known (Opt) Competitive Ratios



Searching with <u>probabilistically faulty</u> agents?

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Linear Search – One p-faulty Searcher

Specs:

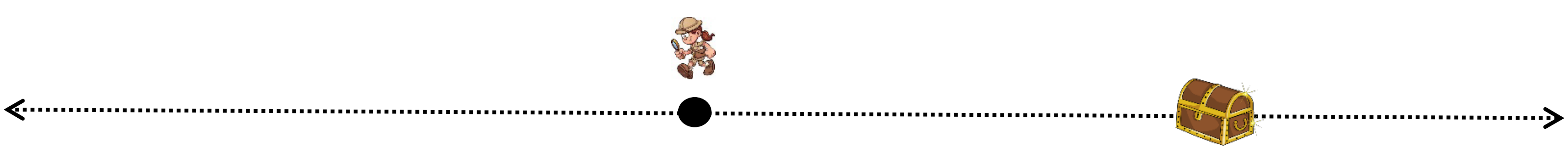
- **One** speed-1 searcher, starting at origin of infinite line.
- No turning cost, no extra time for changing direction.
- Treasure (exit) at unknown location, at least 1 away.
- Treasure cannot be seen from distance.
- Treasure detected when searcher walks over it.
- **Probabilistic faults: p-faulty (specs to be defined shortly)**

Feasible Solutions:

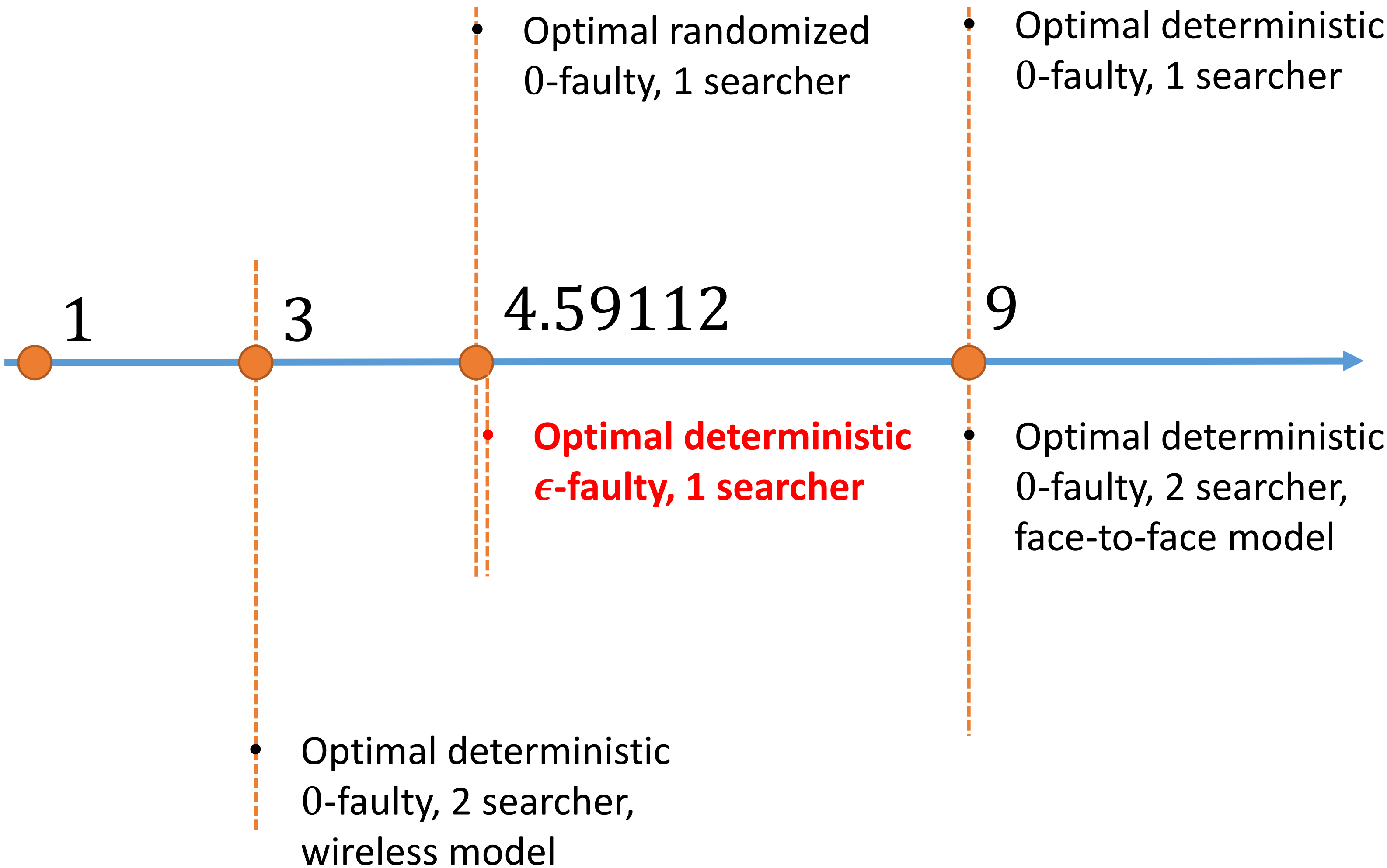
- **(Deterministic or Randomized)** trajectory covering (eventually) entire line.

The objective:

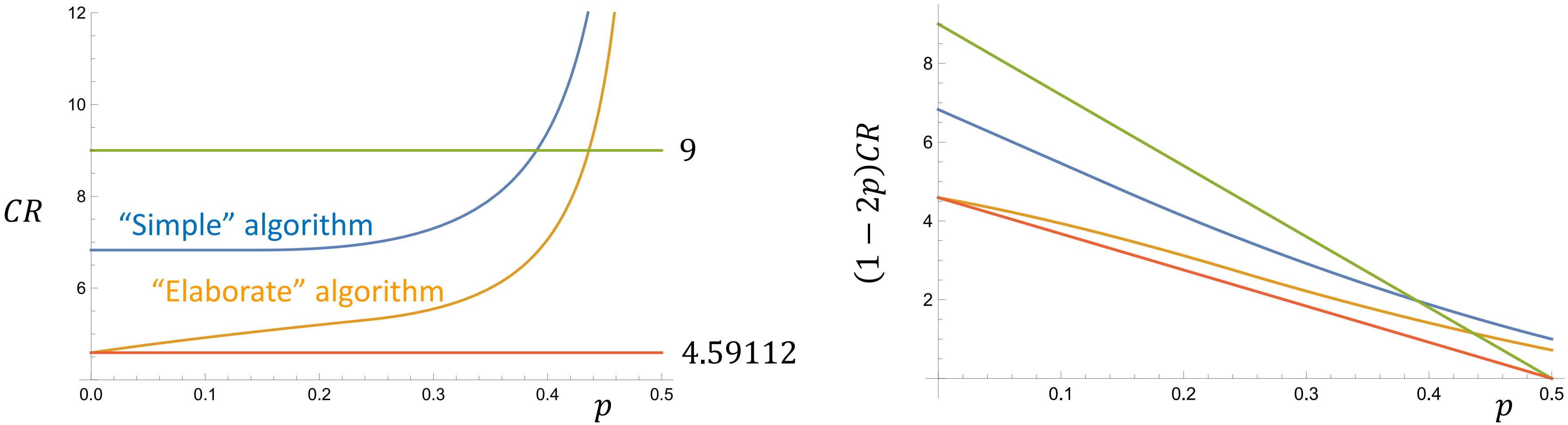
- Minimize "worst case" **expected** relative search time



Known (Opt) & New Competitive Ratios



New Upper bounds
One Deterministic p-Faulty Searcher



Complementary Observations

- No algorithm for one p-faulty searcher has CR better than 4.59112
- Every algorithm for one p-faulty searcher has CR at least $\Omega\left(\frac{1}{1-2p}\right)$

Linear Search – Two p-faulty Searcher

Specs:

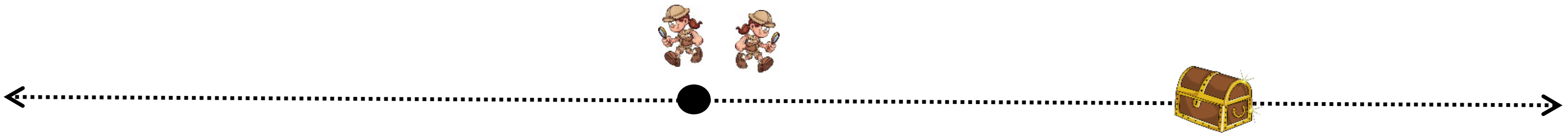
- **Two** speed-1 searcher, starting at origin of infinite line.
- No turning cost, no extra time for changing direction.
- Treasure (exit) at unknown location, at least 1 away.
- Treasure cannot be seen from distance.
- Treasure detected when searcher walks over it.
- **Searchers aware of each others' trajectories (centralized model)**
- **Searchers access to same clock (synchronous model)**
- **Probabilistic faults: p-faulty (specs to be defined shortly)**
- **Communication between Searchers: F2F or Wireless**

Feasible Solutions:

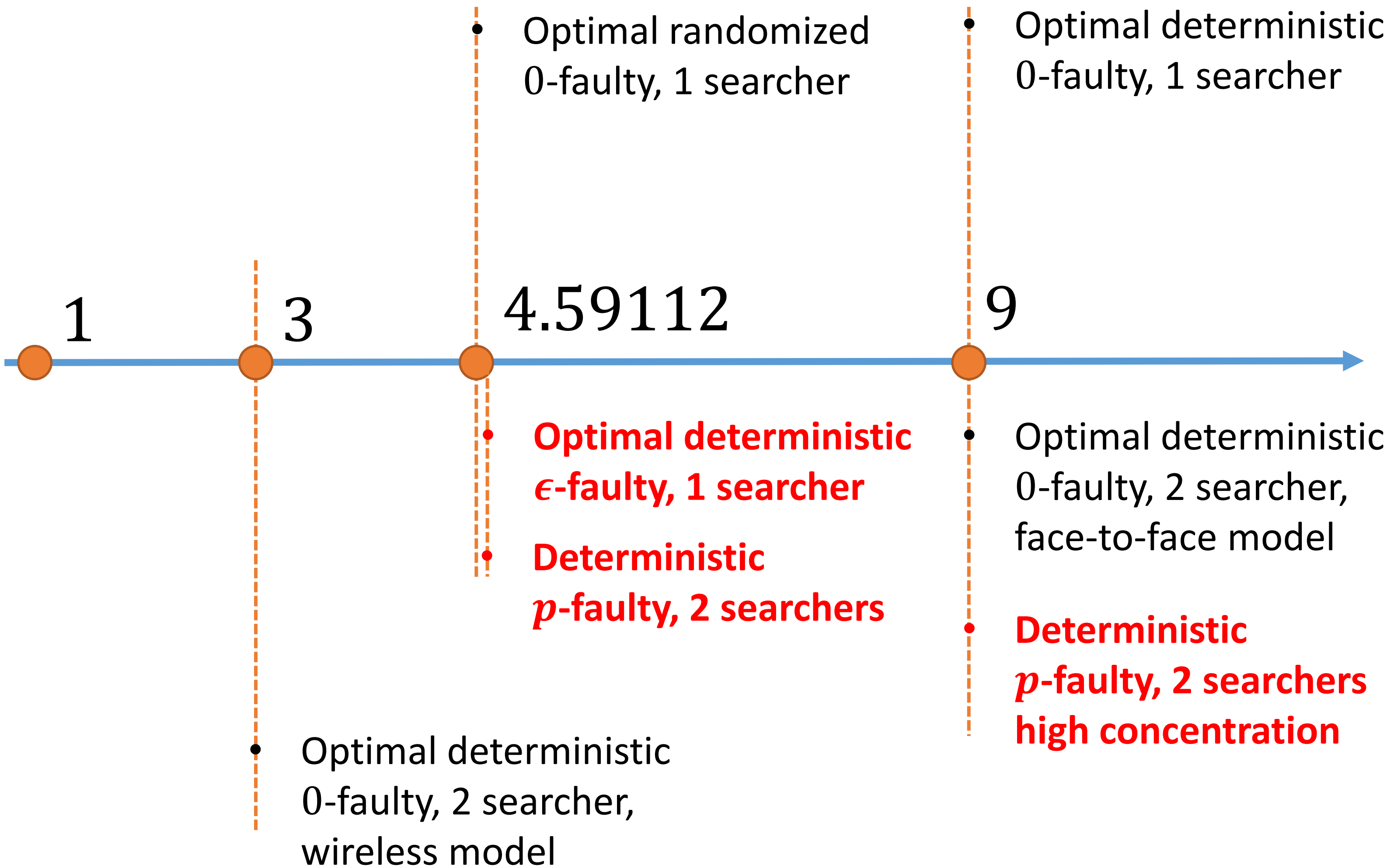
- **(Deterministic or Randomized)** trajectory covering (eventually) entire line.

The objective:

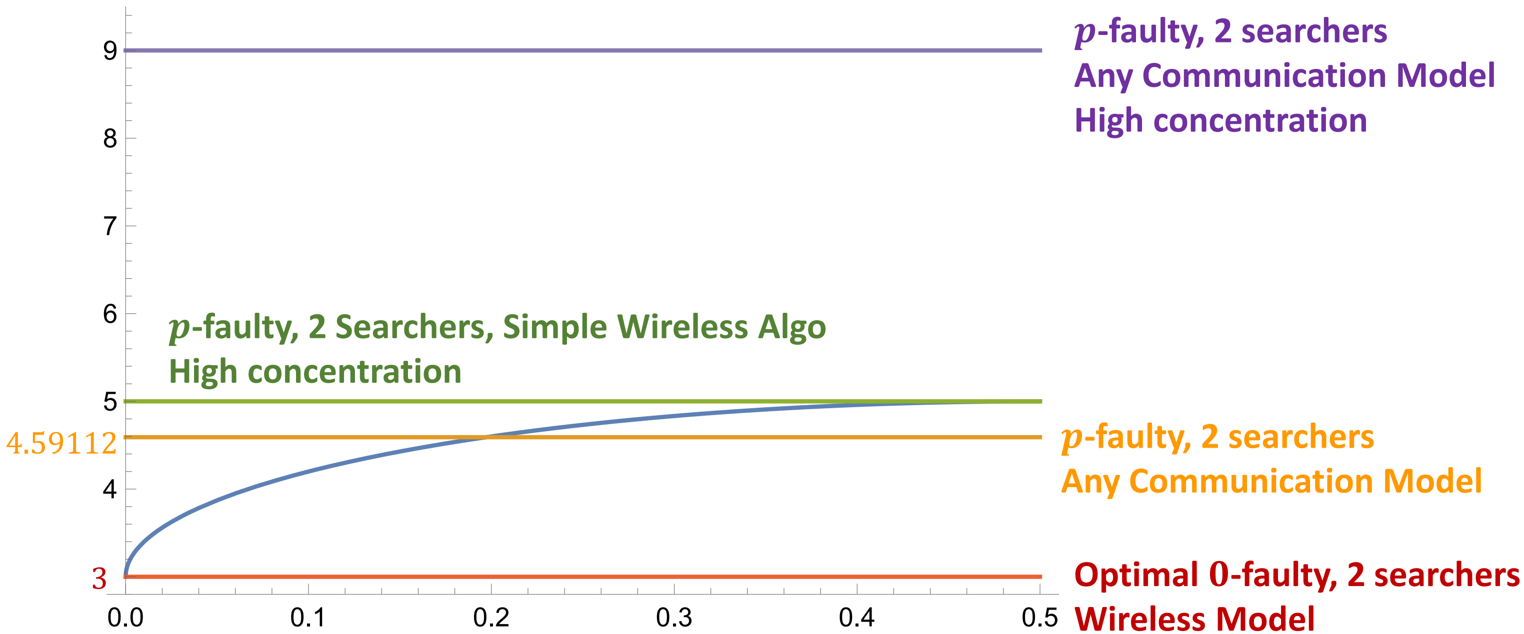
- Minimize "worst case" **expected** relative **evacuation** time



Known (Opt) & New Competitive Ratios



New Upper bounds
Deterministic p-Faulty; Two Searchers



Theorem

Two p-faulty wireless searchers can evacuate with expected competitive ratio

$$3 + 4\sqrt{p(1-p)} + \epsilon,$$

For every $\epsilon > 0$, and with any concentration.

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What Else is Out There?

Rough Summary of Variations

- Number of agents & communication model
  - Wireless, Face-2-face, token based, blackboard, limited range communication
- Feasible solutions, hidden item found by
  - any searcher
  - specific searcher
  - all searchers
- Cost quantification
  - Evacuation of first or last or specific agent,
  - weighted average of termination costs
  - Trade-offs
- Cost Analysis
  - Worst case, average case, competitive analysis
- Domain
  - Line, half-line, k-star, plane, grid, circle, disk, triangles, rectangles, graphs
- Faults
  - Crash faults, byzantine, probabilistic faults
- Other Specs
  - Speed, energy consumption, multiple exits

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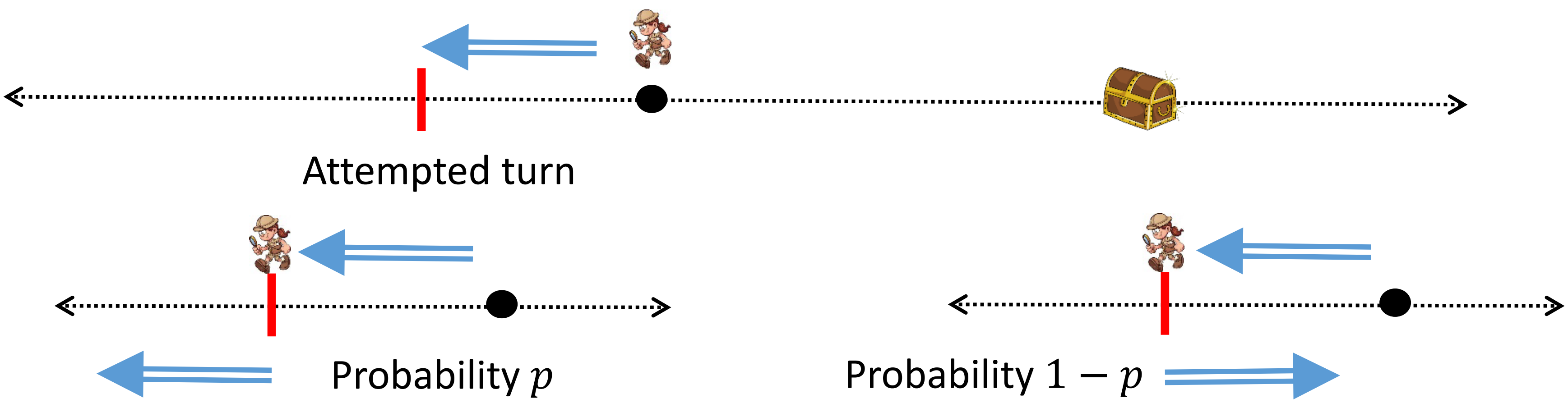
p-faulty Behaviour for 1 Searcher

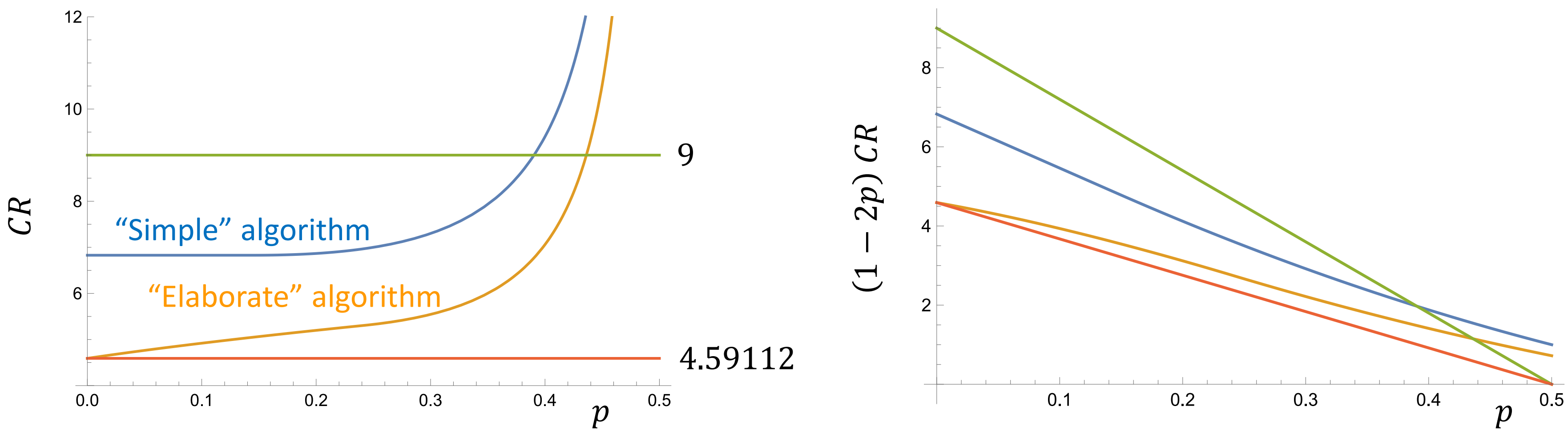
- No orientation
- Every attempt to turn is independent Bernoulli Trial with success $1 - p$.
- Searcher not aware if turn is successful
- Unlimited attempts, whenever.



Anchor Points; can be used for **forced turns**

- Origin
- Treasure (exit)

Reminder: New Upper bounds
One Deterministic p-Faulty Searcher

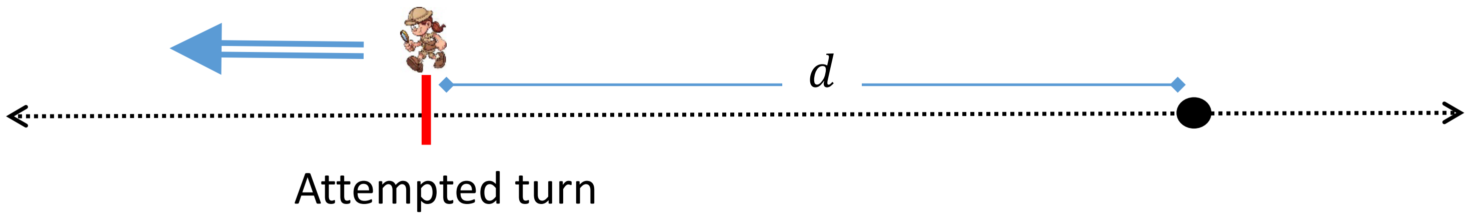


Complementary Observations

- No continuity at $p = 0$
- No algorithm for one p-faulty searcher has CR better than 4.59112
- Every algorithm for one p-faulty searcher has CR at least $\Omega\left(\frac{1}{1-2p}\right)$

Probabilistic Faultiness
A Curse and a Blessing

- **Observation 1**
  - Expected time of returning to origin?
  - Searcher needs $2d$ time to learn if turn was successful.
  - Divergence if $p \geq 1/2$.

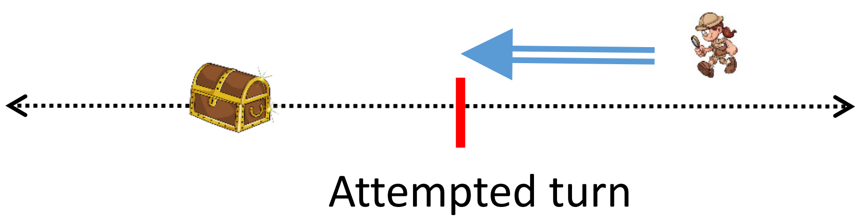


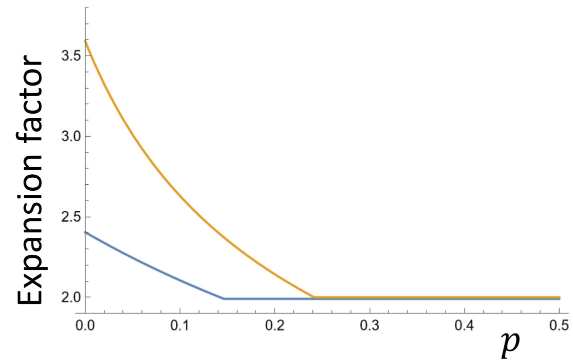
- **Observation 2**
  - Attempt multiple turns close to origin to collect random bits.

  (Similar to forced turns)

Contributions & Technicalities

- Tedious analysis: target found intentionally or unintentionally.
- Zig-zag type algo: Expand by factor g the searched space in each direction.
- Choose expansion factor $g = g(p)$ minimizing CR.





Method for leveraging p-faults into an advantage:

- Arbitrarily close to origin
- Make repeated attempts to turn
- Use origin to determine result of Bernoulli trial; collect random bit

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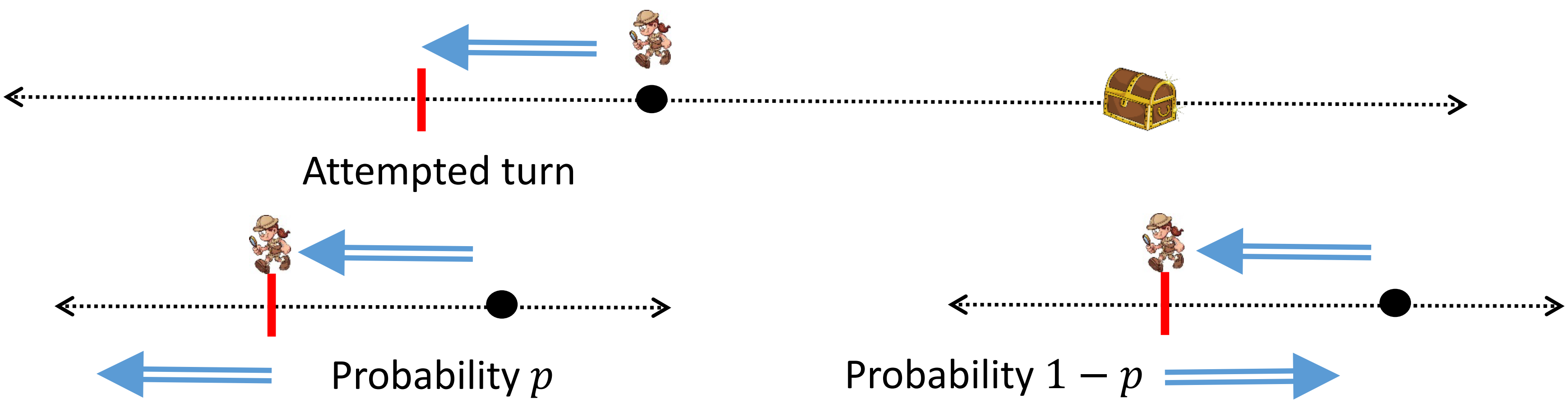
p-faulty Behaviour for 2 Searchers

1-searcher specifications:

- No orientation
- Every attempt to turn is independent Bernoulli Trial with success $1 - p$.
- Searcher not aware if turn is successful
- Unlimited attempts, whenever.



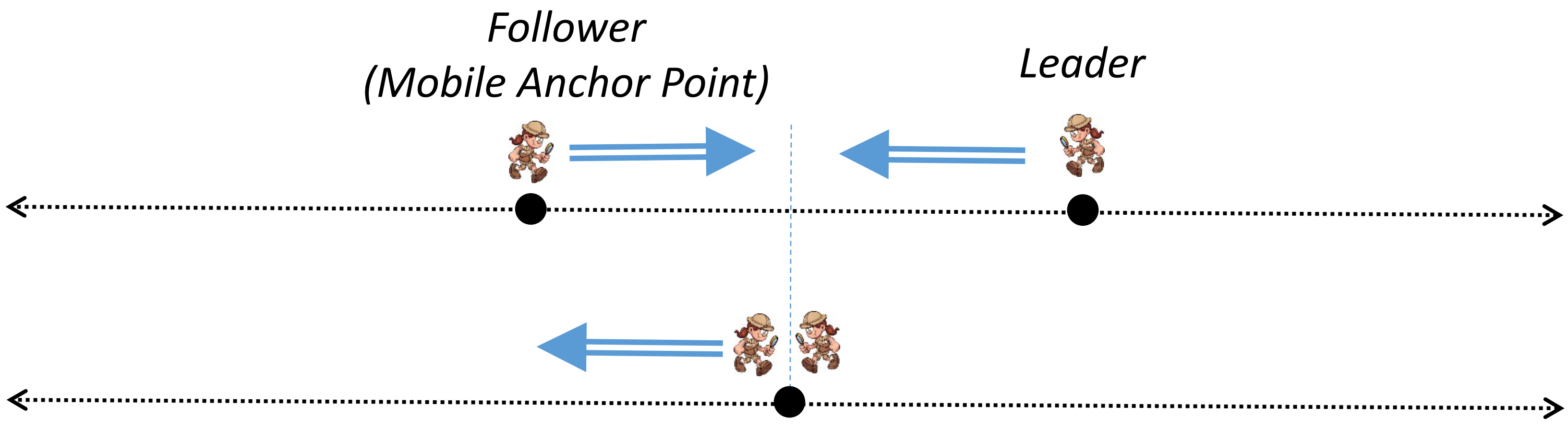
Anchor Points; can be used for forced turns

- Origin
- Treasure (exit)

p-faulty Behaviour for 2 Searchers

<u>2-searcher</u> specifications:

- Searchers can act as anchor points.



<u>Simple Corollaries</u>:

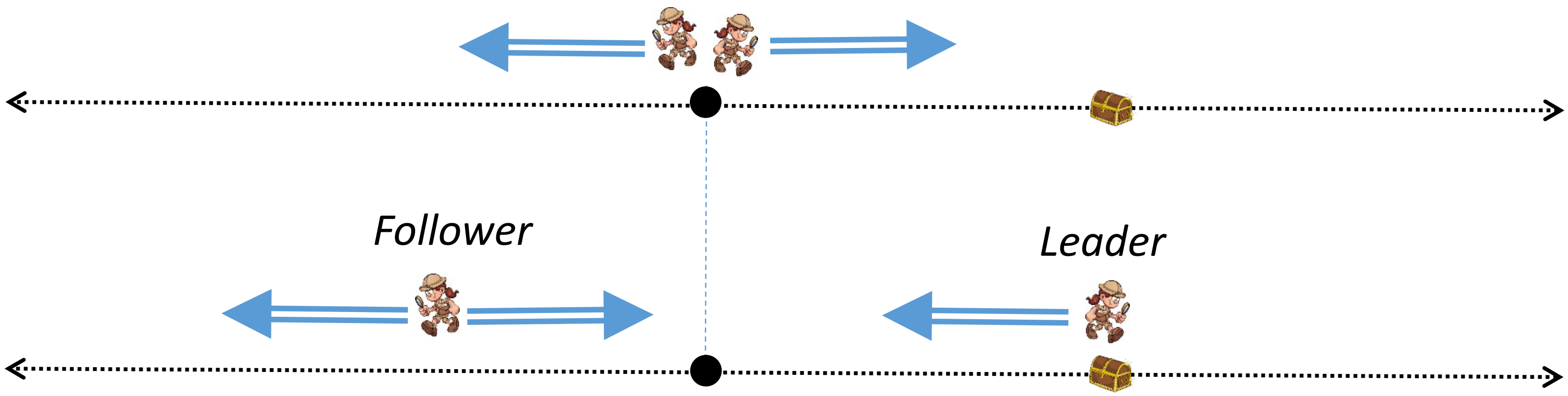
- 2 agents can perform **forced turns**.
- 2 agents can simulate opt 1-agent deterministic algorithm.
- After collecting enough many random bits, 2 agents can simulate opt 1-agent randomized algorithm.

Proof of Theorem

Two p-faulty wireless searchers can evacuate with expected competitive ratio

$$3 + 4\sqrt{p(1-p)} + \epsilon,$$

For every $\epsilon > 0$, and with any concentration.



- Leader performs forced turn using target.
- Follower attempts to turn and switches to speed $s = s(p)$.
- Upon meeting again, agents perform forced turn and return to target.

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Future Directions

- (Better) Lower Bounds for p-faulty agents?
- Other type of faults & domains?
- Communication faults?
- Fleet of searchers & faults.

Thanks!