A Decade of Stone Age Distributed Computing

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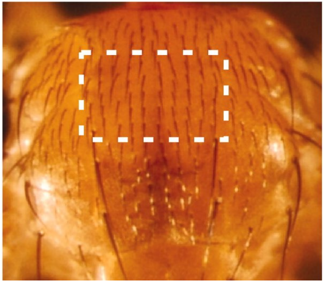
Based on joint works with Yehuda Afek, Eyal Keren, Noa Kolikant, Oren Louidor, Jara Uitto, Roger Wattenhofer

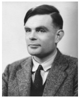
Distributed biological processes

- Social insects, vertebrate colonies, the brain, ...
- **Our focus:** biological processes in cellular networks
  - Abstracted as distributed graph algorithms
- [Afek, Alon, Barad, Hornstein, Barkai, Bar-Joseph 2011]:
  growth of sensory organs in drosophila's nervous system = MIS



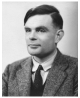
- Cell $\neq$ modern computer
- Aim for networks of "sub-silicon devices"
  - nano-networks, programmable matter, smart paint, smart dust

Sub-silicon devices — communication

- [AABHBBJ]: restricted form of local interactions
- The *beeping* communication scheme
  [Cornejo, Kuhn 2010], [Flury, Wattenhofer 2010]
  - In each (synchronous) round, node beeps or silent
  - Node's signal: 0 beeps or ≥ 1 beeps among neighbors
  - Resembles juxtacrine cell-cell communication
- Special case of *set-broadcast* communication scheme
  [Hella, Järvisalo, Kuusisto, Laurinharju, Lempiäinen, Luosto, Suomela, Virtema 2015]
  - In each (synchronous) round, node broadcasts message
  - Node's signal: set of message types broadcast by neighbors
    - For each message type, distinguish 0 copies from ≥ 1 copies
  - Juxtacrine communication with multiple ligand-receptor types
- What about the node's computational power?
  - [AABHBBJ]: 

Sub-silicon devices — computation

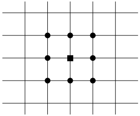
- Computational power of cell?

 $\ll$ 

- [Benenson, Paz-Elizur, Adar, Keinan, Livneh, Shapiro 2001]: cell enzymes “programmed” to implement finite automata
- **Model choice:** abstract  as *finite automaton*
- Distributed computing over finite automata
  - Population protocols
  - Amoebot
  - Cellular automata

Cellular automata

Infinite grid of finite automata

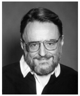


$$q_{t+1}(x, y) \longleftarrow q_t(x, y), \{q_t(x', y') \mid \|(x', y') - (x, y)\|_\infty \leq 1\}$$

Typical question: How initial (finite) configuration evolves?

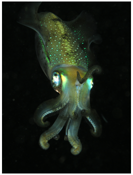
Invented by   (self-replicating systems, crystal growth)

Game of life



Digital physics

Biological processes



Melting pot of model features

- [Emek, Wattenhofer 2013]
  - cellular automata over arbitrary (finite) topologies
  - set-broadcast communication scheme [HJKLLLSV]
  - asynchronous executions
- **Nowadays:** simplified version

Formal definition

- Finite undirected communication graph $G = (V, E)$
- Each node runs *local automaton* over fixed state set Q
  - $q_t(v)$ = state of node $v \in V$ in *step* $t \in \mathbb{Z}_{\geq 0}$
- Signal of node v in step t: $\{q_t(u) \mid u \in N(v)\}$
  - set rather than multiset or vector
- Adversarial asynchronous *schedule* $A_0, A_1, \dots \subseteq V$
  - A_t = subset of nodes activated in step t
- *State transition function* $\delta : Q \times 2^Q \to Q$ (possibly randomized)

$$q_{t+1}(v) = \begin{cases} \delta\left(q_t(v), \{q_t(u) \mid u \in N(v)\}\right), & v \in A_t \\ q_t(v), & v \notin A_t \end{cases}$$

- Function $\omega : Q \to \mathcal{O} \cup \{\perp\}$ determines *output* associated with $q \in Q$
  - $\mathcal{O}$ = task dependent output value set
  - $\perp$ = no output (needed for termination detection)

Uniform algorithms

- Crux of SA model: $|Q|$ and $|\delta|$ are fixed
- Local automaton oblivious to global parameters
  - $n = |V|$
  - $\Delta = \max_{v \in V} \deg(v)$
  - $D = \max_{u,v \in V} \text{dist}(u, v)$
- Sometimes graphs of bounded Δ or D
  - n is always unbounded
- Local automaton of $v \in V$ oblivious to $\deg(v)$
- Truly uniform!

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easier to manufacture

Model variants

- Schedule is *synchronous* if $A_t = V$ for all $t \in \mathbb{Z}_{\geq 0}$
- *Self-stabilization*: adversarial initial configuration $\langle q_0(v) \rangle_{v \in V}$
  - Alternative: *graceful initialization*
    - Alg designer determines initial state $q_{\text{init}} \in Q$
    - $q_0(v) = q_{\text{init}}$ for all $v \in V$
  - **Convention:** graceful initialization unless self-stated stated
- *Termination detection*:
  if $\omega(q_t(v)) \neq \bot$, then $\omega(q_{t'}(v)) = \omega(q_t(v))$ for all $t' \geq t$
  - **Convention:** require termination detection iff graceful initialization
- *Sender collision detection*: G has no self-loops
  - Self-loop on v implies $v \in N(v)$, thus $q_t(v)$ included in v's signal
  - **Notation:**
    - $\text{SA}^{\circlearrowleft}$ = model with self-loops (w.l.o.g. on all nodes)
    - $\text{SA}^{\updownarrow}$ = model with sender collision detection (no self-loops)

Complexity measures

- Runtime a la [Dolev, Israeli, Moran 1997]
  - *Async round*:
    shortest step interval in which each node activated at least once
    - Inductively partition step axis $\mathbb{Z}_{\geq 0}$ into rounds
  - *Runtime of alg* = number of async rounds until alg terminates
  - Sync schedule: each step forms a round
- *State space* = $|Q|$

From synchronous to asynchronous algorithms

Theorem ([Emek, Wattenhofer 2013])

Suppose that a distributed task $\mathcal{T}$ admits a sync $\mathrm{SA}^{\circlearrowleft}$ *(resp.,* $\mathrm{SA}^{\updownarrow}$*) alg that solves $\mathcal{T}$ with state space S and runtime R w.h.p. Then, $\mathcal{T}$ admits an async* $\mathrm{SA}^{\circlearrowleft}$ *(resp.,* $\mathrm{SA}^{\updownarrow}$*) alg that solves $\mathcal{T}$ with state space $O(S^2)$ and runtime $O(R)$ w.h.p.*

Theorem ([Emek, Keren 2021])

Suppose that a distributed task $\mathcal{T}$ admits a sync self-stab $\mathrm{SA}^{\circlearrowleft}$ *(resp.,* $\mathrm{SA}^{\updownarrow}$*) alg that solves $\mathcal{T}$ with state space S and runtime R w.h.p. Then, $\mathcal{T}$ admits an async self-stab* $\mathrm{SA}^{\circlearrowleft}$ *(resp.,* $\mathrm{SA}^{\updownarrow}$*) alg with that solves $\mathcal{T}$ with state space $O(D \cdot S^2)$ and runtime $R + O(D^3)$ w.h.p.*

- Suffices to develop sync algs

Asynchronous unison

- Essence of self-stab synchronizer: *asynchronous unison (AU)* [Couvreur, Francez, Gouda 1992], [Awerbuch, Kutten, Mansour, Patt-Shamir, Varghese 1993]
- Additive cyclic group $\mathcal{K}$
  - $|\mathcal{K}| \geq \Omega(D)$
- Node $v \in V$ outputs *clock value* $\kappa_v \in \mathcal{K}$
- **Post-stabilization conditions:**
  - *Safety*: $u \in N(v) \implies \kappa_u \in \{\kappa_v - 1, \kappa_v, \kappa_v + 1\}$
  - *Liveness*: κ_v incremented $\geq i$ times during any span of $D + i$ rounds
    - increment $=$ $\mathcal{K}$'s $+1$ operation

Theorem

Consider a self-stab AU alg with state space S and runtime R. If task $\mathcal{T}$ admits a sync self-stab alg with state space $S_{\mathcal{T}}$ and runtime $R_{\mathcal{T}}$ w.h.p., then $\mathcal{T}$ admits an (async) self-stab alg with state space $O(S \cdot S_{\mathcal{T}}^2)$ and runtime $O(R + R_{\mathcal{T}} + D)$ w.h.p.

- Holds also for $SA^{\circlearrowleft}$ and $SA^{\updownarrow}$

AU literature

- Problem introduced in [CFG], [AKMPSV]
- Studied further in [Boulinier, Petit, Villain 2004], [Boulinier, Petit, Villain 2005], [Boulinier, Petit, Villain 2006], [Devismes, Petit 2012], [Devismes, Johnen 2019]
  - Various computational models and graph classes
- **General graphs:**

paper	state space	runtime	comments
[AKMPSV]	∞	$O(D)$	
	$\Theta(n)$	$O(D)$	unique IDs
[BPV]	$\Theta(C_G + C'_G)$	$O(C_G + C'_G)$	SA$^{\circlearrowleft}$

  - C_G = minimum longest cycle length among all cycle bases of G (or 1)
    C'_G = length of longest chordless cycle of G (or 1)
    - $C_G + C'_G$ is incomparable to D
- **Goal:** self-stab AU alg with state space and runtime that depend only on D (linear dependency (existentially) unavoidable)

SA to the rescue

Theorem ([Emek, Keren 2021])

There exists a (deterministic) self-stab $\text{SA}^{\circlearrowleft}$ *AU alg with state space* $O(D)$ *and runtime* $O(D^3)$.

Open question: self-stab AU alg with state space $O(D)$ and runtime $O(D)$?

- Relevant outside SA scope

Graceful initialization

Theorem ([Emek, Wattenhofer 2013])

There exists a sync $\mathsf{SA}^{\updownarrow}$ *alg that solves MIS with state space* $O(1)$ *and runtime* $O(\log^2 n)$ *w.h.p.*

Theorem ([Afek, Emek, Kolikant 2018])

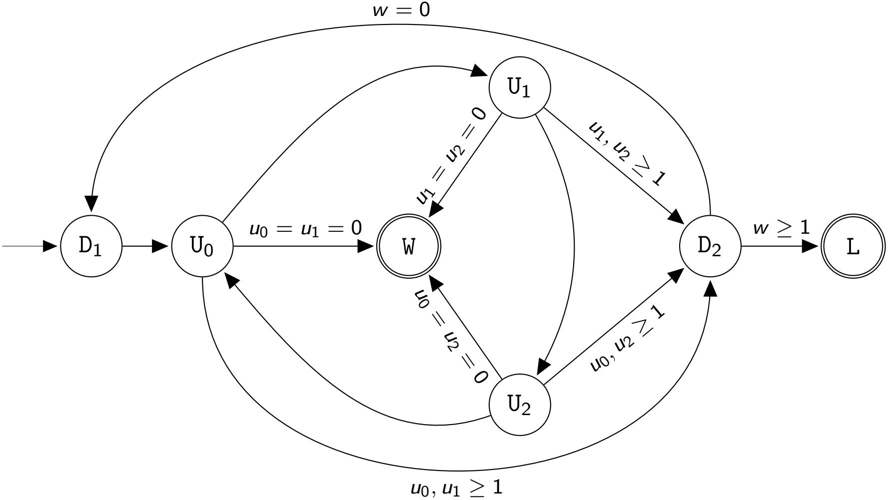
There does not exist a sync $\mathsf{SA}^{\circlearrowleft}$ *alg that solves MIS with state space* $O(1)$ *w.p.* $\Omega(1)$*.*

Theorem ([Kothapalli, Onus, Scheideler, Schindelhauer 2006])

Any anonymous MIS alg with message size $O(1)$ *(including* $\mathsf{SA}^{\updownarrow}$*) requires* $\Omega(\log n)$ *time in expectation.*

Open question: $\mathsf{SA}^{\updownarrow}$ alg that solves MIS with runtime $O(\log n)$?

Local automaton of the SA$^{\updownarrow}$ MIS algorithm



Self-stabilization

Theorem ([Emek, Keren 2021])

There exists a sync self-stab $\mathsf{SA}^{\circlearrowleft}$ *alg that solves MIS with state space* $O(D)$ *and runtime* $O((D + \log n) \cdot \log n)$ *w.h.p.*

- No conflict with impossibility result of [AEK] since
  - D is large in bad graph
  - Requires termination detection
    - Self-stab may be easier!

Self-stabilization — cont.

- [Giakkoupis, Ziccardi 2023]:
  sync self-stab SA$^{\updownarrow}$ MIS alg called *"the 2-state process"*
  - $Q = \{\texttt{IN}, \texttt{OUT}\}$
  - Node $v \in V$ is *satisfied* if either
    - $q_t(v) = \texttt{IN}$ and $\{q_t(u) \mid u \in N(v)\} = \{\texttt{OUT}\}$
    - $q_t(v) = \texttt{OUT}$ and $\texttt{IN} \in \{q_t(u) \mid u \in N(v)\}$
  - If v is satisfied, then $q_{t+1}(v) \leftarrow q_t(v)$
  - If v is unsatisfied, then $q_{t+1}(v) \in_r \{\texttt{IN}, \texttt{OUT}\}$

Theorem ([Giakkoupis, Ziccardi 2023])

The runtime of the 2-state process on $G_{n,p}$ is $\log^{O(1)} n$ w.h.p. if $p \leq \log^{O(1)}(n)/\sqrt{n}$ or $p \geq 1/\log^{O(1)}(n)$.

- Conjecture that runtime upper bound holds for any graph

Fresh from the oven

Theorem ([Emek, Louidor 2023])

There exists a graph on which the (stabilization) runtime of the 2-state process is $n^{\Omega(1)}$ *w.p.* $\Omega(1)$.

- Discussions with Giakkoupis and Haeupler:
  - Holds also for *"the 3-state process"*
    - $\mathrm{SA}^{\circlearrowleft}$ variant of the 2-state process
  - Holds even if $\langle q_0(v) \rangle_{v \in V}$ is chosen u.a.r.

Open question: self-stab SA alg for MIS with state space $O(1)$ and runtime $\log^{O(1)} n$?

Dynamic topology changes

Theorem ([Emek, Uitto 2020])

There exists a sync $\mathrm{SA}^{\updownarrow}$ *alg that solves MIS on dynamic graphs with state space* $O(1)$ *and runtime* $O((C+1) \cdot \log^2 n)$ *w.h.p., where* C *is the number of dynamic topology changes.*

- $O(\log^2 n)$ time amortized over topology changes
- Effect of topology changes is confined:
  "local runtime" of distant nodes is $O(\log^2 n)$ w.h.p.

Theorem ([Emek, Uitto 2020])

There does not exist a sync $\mathrm{SA}^{\updownarrow}$ *alg that solves MIS on dynamic graphs with state space* $O(1)$ *and runtime better than* $\Omega(C+1)$ *w.h.p., where* C *is the number of dynamic topology changes.*

Bounded number of candidates

k-leader selection:
elect a leader out of $\leq k$ (and ≥ 1) initially marked candidates

Theorem ([Afek, Emek, Kolikant 2018])

For every k, there exists a sync $\text{SA}^{\circlearrowleft}$ alg that solves k-leader selection with state space $k^{O(1)}$ and runtime $\tilde{O}(D)$ w.h.p.

Theorem ([Afek, Emek, Kolikant 2018])

There does not exist a sync $\text{SA}^{\updownarrow}$ alg that solves k-leader selection with state space independent of k w.p. $\Omega(1)$.

Self-stabilizing leader election

Theorem ([Emek, Keren 2021])

There exists a sync self-stab $\mathrm{SA}^{\circlearrowleft}$ *alg that solves leader election with state space* $O(D)$ *and runtime* $O(D \log n)$ *w.h.p.*

- No conflict with impossibility result of [AEK] since
  - D is large in bad graph
  - Requires termination detection

Open question: self-stab SA alg for leader election with state space $O(1)$?

Simulating Turing machines

- [EW]: SA$^{\circlearrowleft}$ on path can simulate randomized $O(n)$-space TM
  - TM input encoded in $\langle q_0(v)\rangle_{v \in V}$ "along path"
- How TM input encoded in arbitrary graphs?
  - No natural order among nodes
- Solution: designated *I/O node*:
  - Gets TM input bit-by-bit
  - Returns TM output bit-by-bit

Theorem ([Afek, Emek, Kolikant 2018b])

Any computational task that can be solved by a randomized $O(n)$-space TM with runtime R w.h.p. can be solved by a sync SA$^{\circlearrowleft}$ *alg on bounded degree graphs with state space $O_{\Delta}(1)$ and runtime $O_{\Delta}(R + D)$ w.h.p.*

Wrapping up

- **Take home message 1:** biology through the lens of distributed computing
  - BDA 2023 (co-located with PODC 2023)
- **Take home message 2:** even with (very) weak individual nodes, entire network can be quite powerful
- **Take home message 3:** hard to justify graceful initialization
  - Self-stabilization rocks
    - May be easier (no termination detection requirement)
- Other SA flavors
  - Message passing (async) SA
  - Generalization of set-broadcast to one-two-many counting
  - SA-inspired mobile agents (ANTS problem)
- Additional distributed tasks
  - 3-coloring trees, (α, β)-ruling set, (existentially) small k-dominating set, optimization problems, ...

MUCHAS GRACIAS